Aristotle's Numbers *as* Substances

Abstract

One plausible approach to Aristotle's philosophy of arithmetic is that he held numbers to be hylomorphic substances. While research has been conducted to show the benefits of this theory, particularly in solving the problem of numbers' unity, none has been extensively dedicated to justifying the initial hypothesis that numbers are substances. The absence of such an account is felt acutely due to Aristotle's repetitive negation throughout *Metaphysics* M–N that numbers are not and could not be either forms or hylomorphic compounds. This paper argues that despite these explicit statements, Aristotle does, in a way, consider numbers to be substances. His thinking is grounded in a distinction between numbers' ontological status in the natural world and their mental epistemological mode of being. In the natural world, numbers exist as quantitative properties of things, being numbers just potentially. However, in his understanding, the mathematician separates these properties and considers them as if they were individual entities, i.e. substances. This is how numbers appear to be mental substances by not being actually being such. The paper further clarifies the epistemological procedure which bridges the gap between numbers' dual mode of existence. The so-called method of abstraction allows the mathematician to grasp things as pure abstract units devoid of any sensible qualities except their numerical property. As it is the only characteristic property of the units, it then acts as an element constituting their abstractedness and thus, in understanding, appears as a numerical form tying the units together. This reasoning allows for a substantial account of numbers without committing Aristotle to a theory of numbers as actually independent beings, which he opposes.

Keywords. *Aristotle; arithmetic; ontology of number*

Although Aristotle's views about the ontological status of numbers remain an enigmatic topic and literature is still relatively scarce, two approaches seem to have crystallised. The first group of scholars believes that individual numbers are species of the genus number, with specific cases (e.g. three dogs) being instances of those species. The second argues that numbers are hylomorphic compounds.[1] Neither position is unproblematic. The hylomorphic solution, however, holds

[1] The first approach, with its variations, is advocated by Mignucci (1987), Halper (1989, 261-62), and Katz (2022). Among the supporters of the second theory are Gaukroger (1980), Cleary (1995) and Galluzzo (2018). Pappas's (2018) PhD thesis is a more interesting case: in the first part, it tends towards interpreting numbers as species, but when Pappas reaches the question of unity, he seems to think that Aristotle's answer must be hylomorphism. Finally, he

particular appeal, as it seems to solve one of the most pressing puzzles about number—that of its unity. As Edward Halper famously puts it, "Aristotle reserves the term "heap" for what has little or no ontological status", and that for him, "to be something is *not* to be a heap."[2]

A hylomorphic line of thought accounts for unity by ascribing the unifying role to form: it is not difficult to imagine units as matter, structured by a numerical form. Despite being an appealing solution, this approach was recently seriously criticised by Emily Katz. Her main objection is that "Aristotle's insistence that his opponents account for the unity of number is always conditional upon another of their key commitments: the identification of numbers and substances."[3] This is an identification that, according to her, Aristotle himself does not hold.[4] In a sense, Katz is right. As she proceeds to show, every passage usually quoted in literature indeed has conditional context: *if* numbers are substances, *then* they must have unity. Moreover, Aristotle often explicitly states that numbers are not or could not be substances, i.e. either forms or compounds of form and matter. He denies this from the beginning of *Metaphysics* (see A.9) and repeats it frequently throughout books M–N (e.g. N.3, 1090a29; N.5, 1092b16–25).

In this paper, I argue that there is, after all, a way for numbers to be treated as substances that comes by differentiating its modes of being. Now, in discussions about numbers' mode of being, the text frequently employed is *Metaphysics* book M. At the end of M.2, Aristotle suggests that either mathematical objects do not exist at all (not a viable alternative) or they exist in a certain way that is not unqualified, for the word "exist" has many senses.[5] In M.3, he offers some clarification by proposing that the arithmetician or the geometrician studies mathematical objects by taking that which does not exist separately and positing it as such.[6] It is worth noting that Aristotle does not say that these objects are *actually* separate. But they can be separated in mind through mental activity, the nature of which I will address shortly. Thus, the subsequent question in determining numbers' mode of being seems obvious, yet never explicitly asked in the context of Aristotle's philosophy of arithmetic: what does it mean to be separate (*chôrista*)?

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concludes that how these two different views should be reconciled is unclear. Interestingly, ancient commentators also support hylomorphism; see Mueller (1990).

[2] Halper (1989, p. 256).

[3] Katz (2021, p. 199)

[4] Katz does agree, though, that an ontology that identifies substances with numbers for Aristotle *would* be required to answer the question of unity (2021, p. 200). Furthermore, she does not entirely dismiss the idea that numbers should have some unity, and so in her article, Katz argues for a peculiar kind of unity.

[5] 1077b16–17.

[6] 1078a22–23.

To quote Phil Corkum, "The Greek *chôris* and its cognates, when unqualified, typically in Aristotle refers to the separation that he ascribes to primary substances. (When qualified, the term can refer to other notions, such as local, temporal, or definitional separation.)"[7] Although the theme of separation is complex and controversial and cannot be thoroughly tackled here, the most significant thing is already stated. Separation, first of all, is an *ontological* notion reserved for primary *substances* (i.e. hylomorphic compounds). There is a good basis to believe that, in a way, Aristotelian forms are ontologically separate as well.[8] A conclusion would follow that Aristotle equates being separate with being a substance. In her recent article, Katz makes the same equation, claiming that the conditional "if number is separate" stands for "if number is substance".[9] Yet Aristotle dedicates M.2 to demonstrating that mathematical objects cannot be separate from sensible ones without ontological consequences.[10] Moreover, in M.3 he stressed that their existence is qualified—not unqualified, like substances'. The difficulty is evident, and Katz's disproval that numbers for Aristotle could be independent substances seems sound.

Nevertheless, if the reasoning of this paper is correct so far, *Metaphysics* M.3 imply *considering* numbers as substances. The nuance, then, is indeed extremely subtle and can be easily overlooked: Aristotle differentiates between the actual existence of things in the ontological structure of the world and the epistemological potential of their mental existence. Hence, while numbers do not exist as substances, Aristotle suggests that they appear as such in understanding. Naturally, this requires to clarify the epistemological procedure which should bridge the gap between numbers' dual mode of existence. How do we grasp numbers, if they do not already exist as substances in the natural world?

Vangelis Pappas accurately observes that it is important for Aristotle to highlight mathematics' close ties to the natural world; otherwise, they would not apply to it. Consequently, mathematical objects' existence should be accommodated within those ties.[11] In M.3, Aristotle

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[7] Corkum (2016, p. 2).

[8] For more on this, see Katz 2017. To briefly summarise, the ascription of separate existence to forms was thought to mean separate in thought or definition. Except that this makes *Metaphysics* inconsistent: in various passages where the form is said to be separate, separation has substantial ontological implications (e.g. Δ.8, 1017b23–26; Z.1, 1028a33–34; H.1, 1042a28–31; see Katz (2017, p. 42–52) for a detailed analysis). Therefore, it became necessary to accommodate an ontological meaning of separation to forms to save Aristotle's philosophy from being incoherent. One evident difficulty of such an approach is the lack of textual evidence, namely, Aristotle's direct discussion of the notion of separation. On the other hand, the differentiation of meanings is deduced within Aristotle's theoretical framework, making it probable and convincing.

[9] Katz (2022, p. 205).

[10] See 1076b11–1077b11.

[11] Pappas (2018, p. 75).

stresses that sciences can apply their propositions to sensible objects because these objects have respective relevant properties: this also standss for mathematics. He proceeds to compare the applicability of mathematics with that of physics. Physicists consider their objects *as* moving and do not take into account their nature or other characteristics. It does not follow that there is some moving object separate from sensible substances or that they have a separate moving nature.[12] The same will apply to mathematics—there will be propositions, in arithmetic's case, that could be applied to objects *as* indivisible units.[13] One can conceptualise it as a selective focus, where scientists consider solely the property essential to their scientific field.

Things get confusing later on. Aristotle further writes that if things that the mathematician considers are coincidentally sensible, it does not follow that mathematical sciences are about sensible objects.[14] Nor does it imply, though, that objects of mathematics exist separately from these. The already-mentioned positive answer is presented at the end of M.3—the mathematician takes that which does not exist separately and considers it *as if* it were separate (i.e. as if it were substance).[15] E.g. the arithmetician posits a man as one indivisible, and then studies what is incidental to him as such. Aristotle's train of thought then seems to be as follows: the arithmetician, observing a group of objects, posits them to be indivisible units and then separates their incidental *properties* in understanding as if they were substances. These 'quasi-substances' are the mathematical objects that the mathematician further inquires into.

It can be quite soundly stated that these mathematical objects are obtained by a process of abstraction (*aphairesis*).[16] Although the nature of it has already been extensively discussed, not much consensus (except that this is indeed the way mathematicals are obtained) has been reached. That should not come as a surprise insofar as the questions are interconnected: it is difficult to clarify the method when it is not clear what results one should get by following it. However, in the

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[12] 1077b24–33.

[13] The concept of being an indivisible unit can be puzzling. For instance, in M.3, Aristotle takes a man to be indivisible. Yet it can be argued that a man can be easily divided into parts. Following Pappas's (2018, p. 144) and Galluzzo's (2018, p. 206) interpretation, Aristotle's suggestion here is to consider beings indivisible *per se*, meaning that by dividing a man, one cannot get more men. However, Barnes (1985, p. 114) presents a complicated case that by dividing a cube, one can get more identical cubes. That is why Katz's (2021, p. 211 n. 72) argument seems the most valid. She argues that it is not necessary for a unit to be indivisible as a unit—it is more a coincidence that a man cannot be divided into more men. Bearing in mind that a unit is always some kind of measure, Katz notes how Aristotle writes about defining this measure by taking a thing as one according to the senses (*pros tēs aisthēsin*) (N.1, 1087b37–1088a3). Therefore, while theoretically, a cube can be divided into more cubes, the senses perceive it as one cube, which will be the starting point of further counting.

[14] 1078a3–5.

[15] 1078a18–29.

[16] To name a few in favour of Aristotle's philosophy of mathematics being abstractionistic: Mueller (1970, p. 161), Gaukroger (1980, p. 188), Cleary (1995, p. 312–318), Katz (2022, p. 137).

framework of the 'numbers as substances' hypothesis, we can look at the concept of abstraction once again. It is most explicitly explained in *Metaphysics* K.3, 1061a29–b2. Abstraction appears as a mental procedure where the mathematician not only focuses on what concerns him particularly in sensible objects, but also "leaves" only those qualities, taking away everything sensible. In this case, the mathematician leaves quantity, which is constituted by units and attributes that are intrinsic to it.

To illustrate, let us say there are three people. According to Aristotle's reasoning, when the arithmetician takes away everything that constitutes them as people, he gets abstract units. And as the only thing left characteristic of their abstractedness is the numeric property of being three, one could say that the numeric property is what then *constitutes* the abstractedness. Therefore, after the group of objects' have been reduced just to their quantity, the numerical property appears to act as a form that holds the units (i.e. matter) together.[17] That is how, by grasping the elements that are left after the abstraction, the mathematician performs their mental separation. Moreover, these elements are inevitably thought of in form–matter categories. Stephen Gaukroger aptly names them "noetic mathematical objects" which have form as well as matter, with both components perceived as abstracted and separate in understanding from their sensible counterparts.[18] Of course, the hylomorphic theory has aspects that need further clarifications for it to be well-grounded. Nevertheless, the distinction between numbers' ontological status in the natural world and their mental epistemological mode of being proposed here solves the initial problem of how numbers can be thought of as substances in the first place.

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[17] Cf. M.8, 1084b6–7, where it is said that if number is separate, the units act as a material part, and the number is the form of the units.

[18] That is why Mignucci's reasoning that numbers abstracted from different sensible groups of objects (e.g. ten sheep and ten people) will be different numbers is incorrect. After reducing any quantity to abstract units, the numerical property that constitutes them will be the same.

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