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IMPOSSIBLE WORLDS ARE HERE TO STAY

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SUMMARY: I address objections to impossible worlds (IWs) by Timothy Williamson and Kit Fine. Two species of IWs Mark Jago and I had in our *Impossible Worlds* book were FDE worlds (worlds used in the semantics of the nonclassical logic of First Degree Entailment) and open worlds (worlds not closed under any non-trivial logical consequence relation). Williamson attacks the idea that propositional contents are sets of open worlds; but we explicitly disavowed that very idea. He endorses uses of IWs we developed, except he calls these 'pseudo-worlds'. In *Angellic Content* (AC), Fine champions a truthmaker semantics and in *Constructing the Impossible* advocates its superiority over IWs. But his semantics also includes states that are IWs, as characterized in the literature: representations of absolute impossibilities. They are like Barwise and Perry's situations — which is how FDE worlds have been interpreted for decades. A key difference between AC and FDE is that only the latter validates 'absorption principles' one may find unwelcome in a characterization of propositional content. But a more topic-sensitive FDE can make them fail. Williamson has also objected to the idea of giving truth(making) and falsity(making) conditions separately, as in both AC and FDE. But FDE can work with truth conditions only and the compatibility semantics for negation. AC can work the same way.

KEYWORDS: propositions, content, subject matter, truthmaker semantics, nonclassical logics

RESUMEN: Abordo las objeciones a los mundos imposibles (MIs) formuladas por Timothy Williamson y Kit Fine. En nuestro libro *Impossible Worlds*, Mark Jago y yo presentamos dos especies de MIs: los mundos FDE (mundos utilizados en la semántica de la lógica no clásica del *First Degree Entailment*) y los mundos abiertos (mundos que no están cerrados bajo ninguna relación no trivial de consecuencia lógica). Williamson ataca la idea de que los contenidos proposicionales son conjuntos de mundos abiertos; pero nosotros rechazamos explícitamente esa misma idea. Él acepta los usos de MIs que desarrollamos, excepto que los llama "pseudo-mundos". En *Angellic Content* (AC), Fine defiende una semántica de hacedores de verdad y, en *Constructing the Impossible*, sostiene su superioridad sobre los MIs. Pero su semántica también incluye estados que son MIs, tal como se caracterizan en la literatura: representaciones de imposibilidades absolutas. Se parecen a las situationes de Barwise y Perry, que es justamente cómo se han interpretado los mundos FDE durante décadas. Una diferencia clave entre AC y FDE es que solo este último valida

"principios de absorción" que uno puede considerar indeseables en una caracterización del contenido proposicional. Pero una versión de FDE más sensible a los tópicos puede hacer que dichos principios fallen. Williamson también ha objetado la idea de proporcionar condiciones de verdad (de *truth-making*) y de falsedad (de *falsity-making*) por separado, como ocurre tanto en AC como en FDE. Pero FDE puede funcionar únicamente con condiciones de verdad y la semántica de compatibilidad para la negación. AC puede funcionar del mismo modo.

PALABRAS CLAVE: proposiciones, contenido, dominio temático, semántica de hacedores de verdad, lógicas no-clásicas

> Appeal to impossible states, once they are properly individuated, yields huge dividends.
>
> Kit Fine, *Yablo on Subject-Matter*

1. *From IWs to Topics, and Back*

Following up on the lectures of the 2023 José Gaos Chair at the UNAM, the editors of *Crítica* kindly invited me to inaugurate their Symposia series by providing a target article. Here it is.

Those lectures were called 'Hyperintension without tension. Topics and their applications'. I only talked for a little bit of representing hyperintensional phenomena using impossible worlds (IWs). I was already falling out of love with them, for every few years I like to move on to a new topic and, until recently, this has been *topics*, as per that subtitle.

Topics, or *subject matters*, are what meaningful items, such as sentences, are about. Together with many friends, I've been developing an account of propositional contents — what sentences say or express — inspired by works such as Gemes (1994), Yablo (2014, 2017), that's coming to be called 'two-component semantics' (2C) (see, e.g., Özgün and Berto 2020; Badura 2021; Berto 2022; Ferguson 2023a and 2023b; Özgün and Cotnoir 2025). A proposition is represented by a pair, $P = \langle W_P, T_P \rangle$, where W_P is the set of worlds where P is true and T_P is its topic: what it's about. The account is hyperintensional, in that it gives an individuation of propositional contents more fine-grained than the sets of possible worlds of intensional, standard possible worlds semantics (SPWS). In 2C, sentences true at the same worlds can say different things, express different propositional contents, by being about different things. Of '$2 + 2 = 4$' and 'Equilateral triangles are equiangular', only one is about what two and two add

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up to. Hyperintensional distinctions I used to represent using IWs are aptly modelled using 2C.

The first component in $\langle W_P, T_P \rangle$ has mostly been taken, in the 2C literature, as given by sets of classically possible worlds. But I've argued, again together and/or in agreement with friends (Plebani and Spolaore 2021; Hawke et al. 2024), that truth conditions and subject matter are irreducible in that neither is a function of the other, a view also found in Steve Yablo's wonderful book *Aboutness*, which proposes to "make subject matter an independent factor in meaning, constrained but not determined by truth conditions" (Yablo 2014, p. 2). A 2C spin-off is that one who likes IWs can switch the first component into sets of these, leaving the second untouched. I've quickly introduced you to topics because, later on, we'll combine IWs (of a kind) with topics just in this way.

This paper is about a number of objections to IWs (those I've managed to discuss in a paper of the length recommended by the *Crítica* editors), among the many that have been raised since the publication of a book by Mark Jago and myself, called *Impossible Worlds* (2019, 'F&M' from now on, for *Franz* and *Mark*), a compressed version of which is our entry of the *Stanford Encyclopedia of Philosophy* with the same title (2023, 'the SEP entry', from now on). The objections have been raised by two of the world's greatest living philosophers: Timothy Williamson and Kit Fine, and so, as expected, are very intelligent and interesting.

As we will see, people have been using for decades the label 'impossible worlds' to name beasts of very different sorts. So sometimes an objection, taken by some audiences as being against IWs in general, can actually target only a species of the genus, or rather a particular application of the species. It also happens that, while criticizing some application of IWs (of a kind), people *endorse* things others have called 'impossible worlds' for the same task, except the critics don't use that terminology. Sometimes critics seem to take one species to have been proposed for a certain task, whereas a different one, or none, was proposed for it. So I thought I'd say something to clarify matters on these issues, before possibly kissing IWs goodbye.

The application I'll focus on is: capturing *same-saying*.[1] Most people think that different sentences (or, sentence types) can say

[1] A helpful referee of *Crítica* reminds me to mention that IWs have many more applications, ranging from explanatory reasoning (Kment 2014) to causation and omission (Bernstein 2016), truth in fiction (Badura and Berto 2019), formal (Wansing 1990) and mainstream (Melchior 2021) epistemology, counterpossible conditionals (Nolan 1997), and more. Our F&M book included a survey of applications of IWs.

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(in a given context) the same thing, or express the same content. A traditional name for this is, precisely, 'proposition', or 'propositional content'. One may take it as a task for a semantics, to give an account of propositional content as what is said, thus of same-saying: expressing the same proposition.[2]

That a sentence says something, that two sentences say the same thing, are (ubiquitous) metaphors. Sentences don't say things on their own. People say things, make assertions, using sentences. Most philosophers agree that people can sometimes say the same thing by uttering different sentences:

Arif: Lipa is a kitten.

Midori, *nodding*: Lipa is a young cat.

Arif and Midori said the same thing, because the sentences they uttered differ only by substitution of synonyms.

Same-saying should be, I guess, an equivalence relation. So a semantics giving an account of propositional content (leaving aside complications or extra ingredients required to handle context-sensitive expressions) is to partition the sentences of the target language into equivalence classes of same-sayers. As a rule of thumb: the smaller the cells, the more fine grained the account. How can IWs come into play?

I'll start by rehearsing, in the next section, how the expression 'impossible world' has been used in the literature, drawing on what we already recorded in F&M as well as since the original 2009 edition of the SEP entry. The section after that is about the metaphysical innocence of IWs. This will be quick because nearly everyone, critics included, agrees on IWs being metaphysically kosher. There have almost never been issues with accepting things one may want to call 'impossible worlds' in one's ontology. The issue has almost always been: What are they good for?

Next, section 4 is about a specific kind of IWs, called 'open worlds' since Priest 2005: worlds not closed under any non-trivial (other than $A \vDash A$) notion of logical consequence. Section 5 is about a different

[2] Another helpful referee of *Crítica* asks me what methodology and constraints should govern a good theory of same-saying. This is a crucial question, to which even a sketchy reply may take (at least!) a paper on its own. I'm betting on something along the lines of such a question popping up in some response paper, so I defer my saying something on this to my reply to the responses. Meanwhile, see Ezcurdia and Stainton 2013 for a collection of classic works displaying the many facets of theories of semantic content.

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kind of IWs, called 'FDE worlds' because they have been used in the semantics for First Degree Entailment and relevant logics. Section 6 is about Fine-style truthmaker semantics and compares it with FDE semantics, finding that the former counts as a kind of IWs semantics just as the latter, but highlighting their differences. Section 7 shows how FDE semantics can match Fine-style truthmaker semantics, by being made properly topic-sensitive, in more than one way. Section 8 is about a Williamsonian objection that applies to both Fine-style truthmaker semantics and FDE. Section 9 concludes.

2. *Things People Call 'Impossible Worlds'*

In F&M and the SEP entry we listed four ways of specifying the idea of IWs, ordered from the more to the less general, by recording uses of the expression 'impossible worlds' in the literature. (You'll find lots of references there!)

The first and most general: just as possible worlds are often introduced, in the literature as well as at Modal Logic 101, as *ways things could be or have been*, so are IWs often introduced, in the literature as well as at Nonclassical Logic 101, as *ways things could not be or have been* (see, e.g., Salmon 1984; Yagisawa 1988; Restall 1997; Beall and Van Fraassen 2003). That 'could' is usually taken as expressing absolute or unrestricted modality.

Candidate absolute modalities are often taken to be the logical, e.g., that if snow is white, then snow is white; the mathematical, e.g., that 2+2=4; the metaphysical, e.g., (if you buy the example) that water is H_2O. (People, especially after Kripke, usually group the three under the expression 'metaphysical modality' broadly understood.)[3] I tag along, since critics of IWs often agree on this threefold classification. I stay neutral on whether either of these modalities is reducible to another (e.g. the mathematical to the logical, as logicists want). Mark and I called IWs, so broadly understood, *impossible ways*.

Next, 'impossible world' is often used to label, more specifically, *logic violators*: worlds where some law of logic fails. This depends on what you take the laws of logic to be: given logic L, an impossible

[3] The "maximal objective modality", says Williamson (2024, p. 235). "Necessity in the highest degree", says Kripke (1980, p. 99). And Stalnaker: "We can agree with Frank Jackson, David Chalmers, Saul Kripke, David Lewis, and most others who allow themselves to talk about possible worlds at all, that metaphysical necessity is necessity in the widest sense" (2003, p. 203).

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world with respect to the *L*-laws will be one where some of those laws fail (see, e.g., Hintikka 1975; Priest 2001, 2008a, 2024).

Next, a more specific use names *classical logic violators*: worlds where the laws of classical logic fail (see, e.g., Priest 1997) like a world complying with intuitionistic logic, where Double Negation Elimination can fail.

Next, a still more specific use is for *contradiction-realizers*, like a world where some A is both true and false, or untrue (see, e.g., Lewis 1986; Lycan 1994). Our intuitionistic world will be impossible in the third but not the fourth sense.

Probably because, besides the SEP entry, also F&M was for large chunks a survey of extant literature, these seem to have come to be, for better or worse, the go-to works on IWs (although there are other excellent introductions, e.g., Nolan 2013; Vacek 2023; Tanaka and Sandgren 2024). And the literature has grown *a lot* in recent years. Robert Stalnaker once claimed: "it may be that the best philosophical defence that one can give for possible worlds is to use them in the development of substantive theory" (1991, p. 141). I think IWs are also best defended by their being used in substantive theorizing in hundreds of research works. At the time of my writing, there are over 200 on PhilPapers dealing with IWs, many hundreds listed on Google Scholar. One may complain that most theorizing using IWs is *bad* theorizing, though, and won't just leave the socio-academic success of IWs to speak for itself.

3. *IWs are Metaphysically Harmless*

Most people who accept possible worlds in their ontology will say that one should not take them as merely possible things. Similarly, most realists on IWs will say that one should not take *them* as impossible things. Realists in both camps will claim that these things are, well, real. They are *representations* of things that are (absolutely) possible, respectively, impossible. IWs are a bit like certain Escher drawings: what these represent is (suppose) a way things just could not be.[4] But the drawings are very real.

Except realists on worlds (possible or not) don't take these as concrete things that represent pictorially. Nearly all realists are, to use terminology made popular by critic David Lewis (1986), *ersatzists*: they take worlds to be abstract objects of some kind. Nearly none

[4] I take for granted throughout that there *can* be representations of absolute impossibilities, for this is not generally disputed by critics of IWs, and in particular by Williamson and Fine. For arguments, look e.g. at ch. 1 of F&M.

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is Lewisian, i.e., takes possible worlds to be disconnected spacetimes which represent possibilities by instantiation, or by having the relevant things as parts. A Lewisian world represents the possibility of there being talking donkeys by having as parts real donkeys that talk. Ersatzists take their worlds to represent in some other way.

Admitting IWs of the same kind as Lewisian possible worlds brings troubles we discuss in ch. 2 of F&M (one systematic proposal is Yagisawa 2010's *extended modal realism*; see also Kiourti 2010). But, as noted by Divers (2002) and others, virtually all ersatz realists on possible worlds *already* thereby have, automatically, IWs of the same sort in their ontology: they can't just accept the former and reject the latter. For if possible worlds are maximal possible states of affairs, i.e., states of affairs that could obtain, as in Plantingan realism (1974), then IWs will be (maximal) states of affairs that could not obtain. States of affairs are a bit like propositions (some even identify them with propositions!) in that they can succeed or fail: states of affairs can obtain, or fail to, just like propositions can be true or false; and just like propositions are real even when they're false (and, even necessarily false), so are states of affairs real also when they fail to obtain (and, ditto).

If possible worlds are recombinations of actuality as per various sorts of combinatorial realism (Armstrong 1989), say, rearrangements of the assignments of actually instantiated properties to actual objects, that could be realized (e.g., a world where this actually red chair is blue and this actually blue table is red), then IWs will be rearrangements that could not be realized (e.g., a world where this chair is red and blue all over at the same time).

If possible worlds are maximally consistent sets of sentences, closed under (the right notion of) logical consequence, from a 'world-making' language, as per forms of linguistic ersatzism (Carnap 1947; Adams 1974), then IWs will just be sets of such sentences which can fail maximal consistency/logical closure — sets whose existence is guaranteed by set theory, given that the sets which are the possible worlds exist. And so on. Even an enemy of IWs like Williamson agrees.[5]

IWs can be, trivially, harmless for one who has anti-realist sympathies on possible worlds, say, a modal fictionalist (Rosen 1990), who

[5] "[S]ome philosophers have invoked *impossible worlds*, at which even laws of logic may fail. That can be done without metaphysical outrage, since such a world may simply be identified with a set of sentences of the object-language, treated as the set of sentences true at the world" (2020, p. 243).

thinks there really are no possible worlds other than the actual one: we just make believe that there are. They're a useful fiction to carry out, e.g., modal logic and formal semantics. For one can then avoid commitment to an ontology of IWs by also denying that there really are *such* things, and just talk of useful IWs fictions.

But would the fiction be *useful*? Whether one accepts things called 'impossible worlds' as real, or only takes IWs-talk as fictional talk, the question is always: What is this stuff good for? Critics say: not a lot.

4. *Open Worlds*

Two objections to (applications of) open worlds I want to discuss: that they come with semantics that give disjunctive and, in particular, non-compositional truth conditions; that such accounts are too fine-grained to represent propositional contents, what sentences say or express.

The idea behind disjunctive truth conditions is that, for some operator or other, at possible worlds things go one way and at impossible ones, they go another way. The earliest systematic manifestation of the idea I know of is due to Kripke. In his completeness proof for non-normal modal logics such as C.I. Lewis' systems S2 and S3 (Kripke 1965a), he made Necessitation (when A is a theorem/a logical truth, one gets an extra theorem/logical truth by prefixing a box to it) fail by introducing worlds called 'non-normal'. At them, the modals don't work standardly; rather, all formulas of the form $\Box A$ are stipulated to be false and all those of the form $\Diamond A$ true (intuitively: at such worlds everything is possible, nothing is necessary).

The idea was later generalized to all connectives by Rantala (1982), to deliver modal-epistemic logics modelling non-logically-omniscient agents. At Rantalian IWs, all formulas are assigned a truth value by the valuation function directly, disregarding their logical syntax and treating them as atomic. Graham Priest (2005) called these 'open worlds': worlds not closed under any non-trivial (other than $A \vDash A$) logical consequence. At open worlds, $A \wedge B$ can be true although $B \wedge A$ is not, or although A is not; A can be true although $\neg\neg A$ is not, and so on.

Interestingly, Priest came up with the terminology because he added such worlds to a setting which already had things he called 'impossible worlds': these were certain points of evaluation in the semantics of relevant logic, of which he claimed that they "are to be thought of as (logically) impossible worlds" (2005, p. 15). The idea of

interpreting *such* points as (logically) IWs goes back at least to Priest (1992) if not earlier, and has become canonical after being recorded in Priest (2001, 2008a), the mainstream introduction to nonclassical logics. Such IWs are not 'open', as we'll see later.

Fine's first objection to IWs in *Constructing the Impossible* is from disjunctiveness:

> [W]e would like the compositional clauses for the logical connectives to be 'uniform' or non-disjunctive. This is a theoretical virtue in itself but, without uniformity, it is not even clear that we will have clauses for the logical connectives themselves as opposed to some gerry-mandered product of the theoretician's mind. (2021, p. 141)

Disjunctiveness can't be a problem just on its own (not that Fine thinks otherwise), because there are loads of kosher notions (besides disjunction!) which get a disjunctive characterization: a sibling is a brother or a sister; an Australian was born in Australia or has been naturalized (Priest 2005, p. 237). The supposedly more serious worry is that the disjunctive accounts are not *compositional*.

Compositionality is the principle whereby the meaning of a complex linguistic expression must be a function of the meanings of its constituents, plus the way they have been put together. Only by assuming it, many believe, can we explain how speakers can produce and understand potentially infinitely many expressions starting from finitely many familiar ones. "But impossible worlds 'semantics' sacrifices compositionality" Williamson (2024, p. 122) complains.[6]

However, as Williamson acknowledges, in F&M we came up with a compositionality proof for a semantic account in terms of sets of open worlds (Berto and Jago 2019, pp. 181–184). We relied on a recursive translation between sentences of the object language and 'worldmaking' sentences, whose sets made for open worlds in a linguistic-ersatzist account thereof. For a formula A of the object

[6] I should remark that Graham Priest (2024) takes it as a mistake to assume that a semantics should be compositional if the relevant language is to be learnable. What's required by learnability is that the semantics be at least recursively enumerable, and open worlds semantics of the disjunctive sort can be made to be. After all, also in ordinary SPWS the semantic value of atomic expressions must be fixed independently. "It might be suggested that in any learnable language there should be only a finite number of propositional parameters; if so, in the modal case, one might just restrict oneself to a language with formulas of some finite complexity (which is required, in any case, if the semantics are to be humanly learnable). Alternatively, one may simply restrict oneself to specifications of truth values of formulas at impossible worlds which are recursively enumerable" (p. 351).

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language to be true at one such world w is for the corresponding sentence from the worldmaking language, A^*, to be a member of w. Williamson complains:

> They restore the letter of compositionality by an elaborate construction that involves a hypothetical 'worldmaking' language distinct from the object language under study. Their approach is purely generic, in the sense that it does not depend on any specific features of the object-language operators to which it is applied. This makes it quite uninformative about the object-language. Its hypothetical success consists in *sterilizing* the compositionality constraint, while the constraint's value to semantics had consisted in its fruitfulness. (2024, p. 124)

Now one of the many things on which I've changed my mind with respect to the F&M book (not sure Mark would follow me here), is that I now see the compositionality issue as a red herring. I was enlightened on this by my wise friend, Matteo Plebani. For the very compositionality result we came up with contributes to showing that compositionality sets a low bar.

Take a supremely *coarse*-grained account, say Silly Fregean semantics. In Silly Fregean semantics, sentences only get a Fregean *Bedeutung*. All true sentences express the same content: the True; all false ones express the same content: the False. That can be arranged compositionally, but surely is an inadequate account of content: 'Venice is in Italy' and 'The Earth has one moon' don't say the same just because they have the same truth value.

Take a supremely *fine*-grained account of propositional content — call it Anything Goes — where such contents are represented as sets of open worlds, mapped bijectively to the sentences of the language at issue. That can be arranged compositionally, too, but it makes propositional contents useless, since a key motivation for having contents of this sort was that different sentences should sometimes say the same thing. And surely it is an inadequate account: when 'and' expresses truth-functional conjunction, 'Midori is tall and Arif is thin' and 'Arif is thin and Midori is tall' don't say different things although there are open worlds where $A \wedge B$ is true but $B \wedge A$ is not. In *Constructing the Impossible*, Fine raises precisely such an objection from excessive fine-gaining.[7]

[7] "[I]t is not likely that we will wish to jettison all non-trivial entailments or all non-trivial identities between propositions. Thus even though we may wish to distinguish between the belief in different mathematical truths, it is not clear, in general that we will wish to distinguish between the belief that P & Q and the belief

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Lots of accounts of propositional content in the literature, located strictly between Silly Fregeanism and Anything Goes in fine-grainedness, are compositional. (For one, the SPWS view whereby propositions are sets of possible worlds.) As soon as any of them is past the low compositionality bar, the serious question is: what is the right level of fine-grainedness?

In section 3.3 of Williamson 2024, called "Hyperintensional Semantics; Impossible Worlds", he characterizes the account(s) he is targeting as follows:

> A model contains a set W, whose members are the values of the 'world' parameter with respect to which formulas of the object-language are evaluated as true; W can be any non-empty set. In models for the semantics, the 'impossible worlds' form a proper subset of W; the semantics imposes no constraints on which formulas of the language are true at an impossible world. (p. 119)

So these IWs are open worlds; and the discussion that ensues on pp. 120–129 gives me the impression that he is targeting, among others, the Anything Goes idea that propositional contents, what sentences say or express, be taken as sets thereof.

Anyway, in F&M we explicitly *disavowed* the idea that same-saying could be captured this way. In section 8.4 of chapter 8, called "The Granularity Issue", we claimed:

> [A] particular concept may require a certain kind of world, obeying certain conditions. [...] There are (hyperintensional) notions of *semantic content* which, we think, require us to restrict the domain of worlds. (We discuss one such notion in ∫ 9.6.) These notions require worlds more fine-grained than classical possible worlds, but not as fine-grained as open worlds. [...] None of these cases allow that 'anything goes'. As we shall see in ∫ 9.6, there are substantial, non-trivial equivalences on semantic contents, which we can capture in our impossible worlds framework. (Berto and Jago 2019, p. 178)

And in section 9.6, we came to "*what is said* in the sense of *what is communicated* in making a particular utterance, as opposed to the particular way in which that content is communicated" (p. 206). Here we argued, e.g., that $A \wedge B$ says the same as $B \wedge A$ (examples 9.4a–b); that $\neg\neg A$ says the same as A (9.9a–b); that A says the same as $A \wedge A$

that Q & P [...]. What we need [...] is some intermediate position in which some impossible worlds are allowed and others not" (Fine 2021, p. 142).

(9.10a–b). But, there are open worlds where one from each pair holds and the other fails. So we did not endorse Anything Goes:

> [T]he 'anything goes' approach to content we mentioned in ∫ 8.4 doesn't give an appropriate analysis of same-saying. Some logical relations (including the one relating $A \wedge B$ to $B \wedge A$) preserve same-saying. (2019, p. 208)

What *did* we use open worlds for in F&M, then? In chapter 8 and part of chapter 9, we used them for something Williamson *recommends*; except we developed it in a slightly more detailed fashion. Talking of Fregean puzzles of informative identities, and having used the classic Kripkean furze-gorse and Paderewski cases as running examples, he claims:

> Still, for modelling purposes, we can mitigate the problem in ad hoc ways (see Williamson 2017a for a general discussion of model-building in philosophy). For instance, to see how things look from the perspective of Penny or Pat, we can treat 'furze' and 'gorse' as if they were semantically independent, by allowing metaphysically impossible pseudo-worlds at which they are not coextensive, but which otherwise behave normally. Such worlds may later be epistemically ruled out for the agent by subsequently acquired evidence. That is not a semantic insight, for the words are in fact synonymous, but it does help us understand how Penny and Pat are thinking. Similarly, to see how things look from the perspective of Kripke's Peter, we can work as if there were referentially distinct names 'Paderewski1' and 'Paderewski2', by allowing metaphysically impossible pseudo-worlds at which they do not co-refer, but which otherwise behave normally. Such worlds may later be epistemically ruled out for Peter by evidence he subsequently acquires. That is not a semantic insight, for the names would in fact be synonymous (on a direct reference account, since they are actually co-referential), but it does help us understand how Peter is thinking and predict how he will act. The models enable us to apply the formal apparatus of content-based evidential relations to such cases, in a way which takes account of agents' distorted perspectives on their own contents. (2024, pp. 209–210)

Williamson's 'pseudo-worlds' look like IWs, but I'll give him the terminology. In chapter 8 and part of chapter 9 of F&M, we dealt with things we called 'informational, epistemic and doxastic contents'. I now regret we used the word 'content', for this may have confused readers. On the other hand, those are exactly the pages

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where we claimed that *such* contents should be kept separate from "a distinct notion of informational content, which concerns *what is said*" (p. 188). One might want to relabel them 'pseudo-contents', in Williamson-style terminology. Anyway, we haven't even been the only or first philosophers not to take *content* as a univocal notion. To mention just one, in *On the Plurality of Worlds* Lewis expressed doubts on the idea that one and the same kind of things could play at the same time the role of what sentences say, of the objects of attitudes, the referents of *that*-clauses, the primary bearers of truth values. He famously hypothesized that "the conception we associate with the word 'proposition' may be something of a jumble of conflicting *desiderata*" (1986, p. 54).

We used open impossible (pseudo-)worlds to provide representations of the epistemic and doxastic states of rational, but finite and non-omniscient agents: to see how things look from the perspective of folks like Williamson's Penny and Pat, who lack empirical information on furze-gorse, or on James Newell Osterberg-Iggy Pop (p. 189). We modelled this as the agents having epistemic access to, i.e., seeing as an epistemic possibility, impossible (pseudo-)worlds where 'furze' and 'gorse', or 'JNO' and 'Iggy Pop', are not coextensive. We also came up with formal models of how non-logically-omniscient agents can rule out (pseudo-)worlds by reasoning, after initially seeing them as epistemic possibilities.

Why did we do it? For we thought, as Williamson says, that models of this kind can "help us understand how [one such agent] is thinking and predict how he will act". We in fact endorsed (pp. 162–163) the Stalnakerian picture whereby "Representational mental states should be understood primarily in terms of the role they play in the characterization and explanation of action. [...] [A]ccording to this picture, our conceptions of belief are conceptions of states which explain why a rational agent does what he does" (Stalnaker 1984, p. 4).

And *even* there, we did *not* endorse an unrestricted anything goes approach, whereby any old open impossible (pseudo-)world can be accessible to a rational cognitive agent. For ordinary agents will at least not see obvious impossibilities, like $a \neq a$, or $1 + 1 = 3$, as epistemic possibilities. So we claimed that, for such agents:

> Some contents are not suitable objects of epistemic attitudes. Not all impossible worlds are epistemically possible; some are not epistemically accessible for any agent. [...] Our problem, therefore, is not merely to find worlds not closed under classical consequence. The problem is

> to provide a notion of a world which is logically impossible, but not obviously so. (2019, pp. 191, 195)

This is the purpose for which Hintikka, back in the 70s, had already invoked 'subtly inconsistent' IWs to begin with: worlds which "look possible but contain hidden contradictions" (1975, pp. 476–478). The task is difficult because the distinction between *subtle* and *obvious* impossibilities is elusive, agent-relative, context-relative, and fuzzy. But, in chapter 8 and sections 9.1–9.5 in chapter 9, we came up with an account using open IWs.

Let us now move on to how IWs (of the right kind!) *can* help with propositional content as what is said.

5. *What Is Said*

Take a plain propositional language $\mathcal{L}$ with denumerably many atoms $\mathcal{L}_{AT}$: $p, q, r, \ldots$, negation $\neg$, conjunction $\wedge$, disjunction $\vee$, the usual rules of well-formedness. $A, B, C, \ldots$ are metavariables for its formulas.

Taking propositions as sets of possible worlds, SPWS will assign to each atom p a truth set, $|p|$ — the set of possible worlds where p is true — and truth sets to complex formulas recursively, forming a Boolean algebra of sets: conjunction is set-theoretic intersection; disjunction is union; negation is Boolean complementation; entailment is inclusion.

This is deemed by many (surely by Fine, though not by Williamson) too coarse-grained to capture how what a sentence says can include, or be the same as, what another sentence says. That Arif is tall (classically) entails that either Arif is tall or extremally disconnectedness is no hereditary property of topological spaces. But when Midori says, 'Arif is tall', it doesn't look like she has thereby also said that either Arif is tall or extremally disconnectedness is no hereditary property of topological spaces. Sure, that follows (classically) from what she said. It doesn't seem she has said *that*, though.

The identification of content as what is said with sets of possible worlds clashes with what Steve Yablo has called "our sense of when sentences say the same thing" (2014, p. 2): 'Equilateral triangles are equiangular' and '2 + 2 = 4' seem to say different things: only one is about what two and two add up to. Same-saying should be sensitive to what sentences are about. More precisely: it should imply coincidence in subject matter. One motivation for truthmaker semantics, Fine-style, is to better capture subject matter in a more fine-grained setting than SPWS: see, e.g., Fine 2020.

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Surely we don't want to get as fine-grained as the syntax of the language, or there would be no point in having a semantics. If a Boolean algebra of propositions is too coarse-grained, can one come up with a more fine-grained but not too fine-grained algebra?

One may take contents as given by sets of circumstances more fine-grained than classical possible worlds; but which, unlike anarchic open worlds, still display some logical closure — like the situations of *situation semantics*, as per Barwise and Perry 1983; Kratzer 2021. These may be thought of as parts of reality which don't take a stance on each A: the rainy situation in St Andrews makes true 'It's rainy in St Andrews', makes false 'It's sunny in St Andrews', but is silent on whether or not it's sunny in Melbourne. Situations could also be taken as abstract information-carriers (Barwise and Seligman 1997) which may, on occasion, represent inconsistent infomation.

So situations have been used to make sense of the points of evaluation in the semantics of First Degree Entailment (FDE), a logic which has been studied by Mike Dunn (1976) and Nuel Belnap (1977), and whose frame semantics goes back at least to Routley and Routley (1972).

FDE is a simple paraconsistent (and paracomplete) logic: one whose notion of logical consequence fails the Explosion principle: $A \wedge \neg A$ will not FDE-entail arbitrary Bs (and, $A \vee \neg A$ won't be FDE-entailed by arbitrary Bs). It does so by hosting points which can be partial as well as inconsistent, without making true all formulas of the language.

In the unfortunate case that dialetheism is mistaken, inconsistent points represent ways things could not be or have been. They then count as impossible worlds in the broadest sense of *impossible ways*. They count in fact as impossible worlds in the most restrictive sense, namely *contradiction-realizers*.

One way to present the semantics for FDE is as a four-valued functional semantics. A model is a pair $\langle W, v \rangle$, where W is a nonempty set of FDE worlds. A valuation v at each world w assigns to each atom p in $\mathcal{L}_{AT}$ one of the values **T**rue, **F**alse, **B**oth true and false, **N**either true nor false.[8] A world w makes true an atom p ($w \Vdash p$) when $v_w(p) \in \{\mathbf{T}, \mathbf{B}\}$; makes it false ($w \dashv\!\Vdash p$) when $v_w(p) \in \{\mathbf{B}, \mathbf{F}\}$. This is extended to the whole language via the recursive clauses:

- (S¬+) $w \Vdash \neg A$ iff $w \dashv\!\Vdash A$

[8] Another way is to use a two-valued relational valuation: formulas can be related at worlds to the **T**rue, the **F**alse, both or, neither: see Priest 2008a, ch. 8.

- (S$\neg-$) $w \dashv\!\mid \neg A$ iff $w \mid\!\vdash A$
- (S$\wedge+$) $w \mid\!\vdash A \wedge B$ iff $w \mid\!\vdash A$ and $w \mid\!\vdash B$
- (S$\wedge-$) $w \dashv\!\mid A \wedge B$ iff $w \dashv\!\mid A$ or $w \dashv\!\mid B$
- (S$\vee+$) $w \mid\!\vdash A \vee B$ iff $w \mid\!\vdash A$ or $w \mid\!\vdash B$
- (S$\vee-$) $w \dashv\!\mid A \vee B$ iff $w \dashv\!\mid A$ and $w \dashv\!\mid B$

Negation flips truth and falsity; a conjunction is true iff both conjuncts are, false if either is false; dually for disjunction.

Five remarks: first, the semantics is obviously compositional. Second, the truth and falsity conditions are not disjunctive: we give them uniformly across all FDE worlds. Third, one gets full-fledged *relevant logics* (accounts in the broad family of *substructural logics* (Restall 2000; Paoli 2002), and which formalize a notion of relevance for conditionality and entailment), out of FDE, by adding a relevant conditional, $\rightarrow$. In the mainstream Routley-Meyer treatment (1972a, 1972b, and 1973), its semantics comes via a ternary relation R between points: $w \mid\!\vdash A \rightarrow B$ iff for all Rww_1w_2, if $w_1 \mid\!\vdash A$, then $w_2 \mid\!\vdash B$.[9]

Fourth, when relevant logicians came to the issue of how to interpret this kind of semantics, a popular reply was to understand the points in the frames as information-carrying structures and information conduits, precisely by reading them as the situations of situation semantics à la Barwise and Perry 1983; Barwise and Seligman 1997. This interpretation of the worlds of FDE and relevant logics has been around at least since Restall 1995b; Mares 1997; see also Mares 2004.

Fifth, the semantics recursively assigns to each formula A a pair of a truth $|A|^+$ and falsity $|A|^-$ set (a set of FDE worlds making A true and, respectively, false). Setting the propositional content of A, what A says, as $|A| = \langle |A|^+, |A|^- \rangle$, this delivers a De Morgan algebra of contents: a bounded, distributive lattice where negation-complementation is an involution satisfying De Morgan's laws. We're on a better route than Anything Goes, for we now have it that, when $\bowtie$ ('junction') is $\wedge$ or $\vee$, $|A| = |A \bowtie A|$ and $|A \bowtie B| = |B \bowtie A|$; also $|\neg\neg A| = |A|$, $|\neg(A \wedge B)| = |\neg A \vee \neg B|$, etc. On the other hand,

[9] In the four-valued setting, one needs to give falsity conditions as well, and these can get cumbersome (Restall 1995a). So a more popular approach is to give a bivalent (functional) semantics with truth conditions only. I have postponed this version of FDE, for reasons that will become clear later. (*Spoiler warning*: I think truth conditions only are *better* than truth-and-falsity conditions.)

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$|A \bowtie \neg A| \neq |B \bowtie \neg B|$, (what A is about needn't have to do with what B is about), etc.

Does this do a good job at capturing same-saying? One issue, which Mark and I raised in section 9.6 of our F&M book, is with the pair of principles often called Absorption. FDE semantics gives us that $|A| = |A \wedge (A \vee B)|$ and $|A| = |A \vee (A \wedge B)|$. For an account of same-saying, this sounds bad. When Midori says, 'Arif is tall', it doesn't seem she has thereby said that Arif is tall and either he is tall or extremally disconnectedness is no hereditary property of topological spaces. Fine, Yablo, and others agree that what has gone wrong here is that FDE fails to capture subject matter properly: 'A and/or (A or/and B)' is partly about whatever B is about, which may have little to do with what A is about.

But, Absorption principles make for weak algebraic identities: $a = a \vee (a \wedge b)$ and $a = a \wedge (a \vee b)$ hold in any lattice (any ordered set where least upper bounds $a \vee b$ and greatest lower bounds $a \wedge b$ exist for each a, b). This includes non-distributive and non-modular ones, less structured than a De Morgan algebra. So finding the right structure for same-saying is not going to be easy. However, truthmaker semantics à la Fine does a good job here, so we ourselves started to talk of truthmaker semantics at this point, in 9.6 of F&M (Berto and Jago 2019, pp. 210–211), and our discussion was somewhat inconclusive there. Let's now delve into (a version of) Finean truthmaking.

6. *AC vs FDE*

Fine's attitude towards the *general* idea of IWs as representations of the impossible is bound to be different from Williamson's. For Fine is a hyperintensionalist, in the sense of one who endorses an account of same-saying strictly more fine-grained than that of merely intensional SPWS. He's also a hyperintensionalist in metaphysics and a main figure in the so-called 'hyperintensional revolution' (Nolan 2014), being one of the founding fathers of the grounding movement; and grounding is generally, if not completely uncontroversially, taken as a 'worldly' hyperintensional notion.

Still, the devil is in the details. I'll stick essentially to a presentation given in the "Angellic Content" paper (Fine 2016) (AC), though I will also refer to the "Constructing the Impossible" paper (Fine 2021), for that's where objections to IWs come up, as we have seen.

Same language $\mathcal{L}$ as before. A model is a triple $\langle S, \sqsubseteq, v \rangle$, where S is a nonempty set of *states* and $\sqsubseteq$ is a partial order representing

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mereological inclusion. States can stand in non-trivial mereological relations, e.g., the state of St Andrews being rainy is a proper part of the state of St Andrews being rainy and windy. Out of $\sqsubseteq$ one can define fusion $\oplus$ as usual. In the AC setting, this is complete: any $S_1 \subseteq S$ has a least upper bound, the fusion of the items in S_1.

A valuation v assigns to each atom p a pair of nonempty sets, one of verifiers $|p|^+ \subseteq S$ and one of falsifiers $|p|^- \subseteq S$, so $v(p) = |p| = \langle |p|^+, |p|^- \rangle$. A state s truthmakes an atom p ($s \Vdash p$) when $s \in |p|^+$, falsemakes it ($s \dashv\Vdash p$) when $s \in |p|^-$. This is extended to the whole language via recursive clauses quite close to those of Van Fraassen 1969's "Facts and Tautological Entailment" paper:

- (S$\neg$+) $s \Vdash \neg A$ iff $s \dashv\Vdash A$
- (S$\neg$−) $s \dashv\Vdash \neg A$ iff $s \Vdash A$
- (S$\wedge$+) $s \Vdash A \wedge B$ iff for some t and u, $t \Vdash A$ and $u \Vdash B$ and $s = t \oplus u$
- (S$\wedge$−) $s \dashv\Vdash A \wedge B$ iff $s \dashv\Vdash A$ or $s \dashv\Vdash B$
- (S$\vee$+) $s \Vdash A \vee B$ iff $s \Vdash A$ or $s \Vdash B$
- (S$\vee$−) $s \dashv\Vdash A \vee B$ iff for some t and u, $t \dashv\Vdash A$ and $u \dashv\Vdash B$ and $s = t \oplus u$

A consequence of completeness and the way the semantics is set up is that any formula of the language has some truthmaker, including contradictions $A \wedge \neg A$ (and, any has some falsemaker, including classical tautologies like $A \vee \neg A$):

> It should be noted that this result means that the state space $(S, \sqsubseteq)$ should be taken to include 'impossible' states that will verify such formulas as $p \wedge \neg p$, since a verifier for p and a falsifier for p will fuse to give a verifier for $p \wedge \neg p$. [...] [I]t turns out to be essential to allow impossible states in providing an adequate semantics for Angell's system. (Fine 2016, p. 206)

Three remarks: first, Finean states look a lot like the worlds of FDE semantics (Silva (2025) has a nice discussion of this). Like FDE worlds, they can be inconsistent, both truthmaking and falsemaking A, or truthmaking both A and its negation. Again in the unfortunate case that dialetheism is mistaken, such Finean states, thus, are *impossible ways*. They are impossible worlds in the broadest sense

of 'impossible world', as used in the literature and singled out in section 2 above, i.e., representations of absolute impossibilities: ways things could not be or have been, where 'could' expresses absolute or unrestricted modality. They are impossible worlds in particular in the narrowest sense, i.e., *contradiction-realizers*. Like FDE worlds, they can be partial, neither truthmaking nor falsemaking A, or truthmaking neither A nor its negation.[10] Like FDE worlds, states can recapture classically possible worlds as the special case of points that are both consistent and maximal.

Second, Fine's states are very much like situations à la Barwise-Perry, i.e., what relevant logicians have invoked, as we have seen, to make sense of the points of evaluation in the semantics of FDE and relevant logics, possibly even before Fine-style truthmaker semantics was systematically developed.

As Fine (2017a) masterfully reconstructs the history of truthmaker semantics in his introduction to the topic, the *crucial* point of distinction is: within the situation semantics tradition, the Barwise-Perry-Krazter approach has focused on inexact verification, whereby a situation or state need only be partially relevant for the sentence it makes true; this is also what goes on in FDE above. Instead, exact verification, what Fine focuses on following Van Fraassen (1969), requires full relevance: the rainy St Andrews situation is an exact verifier, but the rainy and windy situation is an inexact verifier, for 'St Andrews is rainy'.

Third, setting the propositional content of A, what A says, as $|A| = \langle |A|^+, |A|^- \rangle$ in AC,[11] the most important difference between the FDE and AC accounts of same-saying is, I think, that because

[10] When Fine jokes that '[t]he possible worlds approach is fine but for two features: the first is that possible worlds are worlds, i.e. complete rather than partial; and the second is that they are possible' (2017b, p. 645), one has to give him the terminological point that calling partial points 'worlds', as IWs theorists have long done, e.g., while talking of FDE, is misleading, insofar as the idea of world evokes that of maximality. But then, many things are ill-named ('counterfactuals') and will stay so because usage is entrenched. *C'est la vie*.

[11] Things are actually richer than this, because Fine can define various notions of content by playing with closure conditions on sets of states. E.g., in a *unilateralist* conception (where one only looks at truthmakers), the *exact* content of A is $\{s \in S : s \Vdash A\}$; the *complete* content is the complete closure of the exact; the *replete* content is the convex closure of the complete. Fine's paper has the title it has, because he proves that the first degree fragment of a logic of analytic containment developed by Angell in the Seventies is sound and complete with respect to the AC truthmaker semantics, in the sense that the equivalence of A and B is provable in the logic iff A and B have the same replete content in all models (see sections 6–9 of Fine 2016).

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Fine gives truth(making) and falsity(making) conditions differently from those of FDE and following those of Van Fraassen's, AC invalidates Absorption: $|A|$ is not, in general, $|A \vee (A \wedge B)|$ or the dual. This is because it may happen that no truthmaker of A includes any truthmaker of B, no falsemaker of A includes any falsemaker of B. Levin Hornischer (2020) has an extensive discussion of this feature; see also Hawke et al. 2024.

To sum up: we have established that AC semantics *is* a kind of impossible worlds semantics, if we stick with common usage of the expression 'impossible world'. However, one could say that, even conceding all the merely terminological points (what we may now call), 'AC-impossible worlds semantics' does a better job than 'traditionally so-labeled impossible worlds semantics', as in FDE-relevant logics, when it's about capturing same-saying, because of the story with Absorption. I'll come to this in the next section.

Before we get there, I should mention that, in "Constructing the Impossible", Fine has other critical remarks on IWs besides the two objections we have already dealt with, namely the one from disjunctiveness and the one from fine-graining. While those two clearly targeted open worlds semantics, the further remarks might be taken as targeting FDE semantics, too.

In that paper, we have an initial state space $\langle P, \sqsubseteq \rangle$, intuitively comprising only *possible* states, and only bounded completeness, rather than completeness, is imposed, i.e., only the $P_1 \subseteq P$ that have an upper bound have a least upper bound. The idea is that not any state can be fused with any other, e.g., the state of this table being round and that of this table being square; for this would deliver an absolute impossibility.

Won't we sometimes miss the fusions specified in the truth/falsity(making) conditions (S$\wedge$+), (S$\vee-$) above? Fine extends the state space to make room for the impossible, allowing states that are fusions of absolute incompatibles — e.g., one where this table is round and square. Impossible states would be something like 'virtual fusions' on sets of states that wouldn't have one otherwise. Fine shows (and claims) that this is as natural a construction as extending the space of rational numbers, filling its gaps with the irrationals, to get the reals. We let the possible be our guide to the impossible. He compares this, favourably, with a procedure whereby we "countenance separate clauses for truth and falsehood under the possible worlds semantics" (pp. 148–149) and we come up with an FDE-style semantics.

That of constructing the impossible à la Fine is a wonderfully clever idea, and the setting Fine comes up with, using ideals, is elegant and spotless for its purposes. But, first, I think fans of IWs of various (other) kinds could fruitfully adopt aspects of the construction as well. That a partial ordering or a fusion operation on worlds does not show up in FDE as presented above, doesn't mean that it can't be added. It *has* been added, decades ago: various semantics for relevant logics embed an information-inclusion ordering $\sqsubseteq$ between their points (see e.g. Dunn and Restall 2002; Priest 2008a; Berto 2015 for introductions to the idea), similar to the one of the Kripke (1965b) semantics for intuitionistic logic. '$x \sqsubseteq y$' means that y supports at least all the information in x. When non-maximal points are around, they can stand in non-trivial information-inclusion relations, as again we know from situation semantics.

Second, the idea may have limited applicability anyway, for, as Fine acknowledges, it seems that not all the hyperintensional distinctions we may want to represent can be delivered by constructions which are fusions of incompatibles, having only old possibilities as their base. We may want to tell apart Hesperus' being self-different from Muhammad Ali's being self-different, for instance, for only one concerns the boxer (and, see Berto 2022, pp. 47–48, for reasons why a truthmaker theorist may need impossible states that serve as falsifiers for claims like '2 is a number', etc.).

We should then also let in what Fine calls 'modal monsters' (p. 155): states which are impossible, not because of being made of incompatible possible parts. The final section 9.5 of the paper delves into this, and I think, again, the construction proposed there may be used to fertilize IWs theories of various kinds — with thanks to Fine.

7. *Topic-Sensitive FDE*

Absorption fails in AC because, as one can prove by induction, when A and B are AC-equivalent, they must have the same atoms. So one obvious idea to make FDE invalidate Absorption is to supplement it with (what has been called) an atom-based account (Hawke 2018) of subject matter. 2C semantics gives one such simple account. The idea is not completely new either: Francisca Silva (2024) combines FDE with topics already, though she goes for an account of subject matters based on partitions of modal space, in the style of Lewis (1988).

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In 2C, remember, a propositional content is a pair $P = \langle W_P, T_P \rangle$, where W_P is the set of worlds giving truth conditions (a 'thin proposition', as Yablo 2014 has it); T_P is the topic or subject matter (making of P a 'thick' or 'directed' proposition: one that points at, or is directed to, what it's about). Now take the worlds at issue to be FDE worlds. Supplement FDE models with a complete join semilattice $\langle T, \oplus \rangle$: T is a set of topics; $\oplus$, topic fusion, given any two topics x and y in T, lumps them together into $x \oplus y$.

A topic function, t, assigns a topic in T to each atomic formula to begin with. Which topic? Any assignment will do: when we are interested in the logic of same-saying, we don't care. (Just as, in truthmaker semantics, we don't care about which verifiers and falsifiers are assigned to the atoms.) We do care about not copying the syntax, surreptitiously taking atomic formulas to be their own topic. This is secured by the fact that it can happen that $t(p) = t(q)$, for different p and q: distinct atoms can talk about the same things.

Otherwise, we only care about how complex sentences turn out to say the same, due to their logical form. Topics are then assigned recursively to them: $t(\neg A) = t(A)$ ('Snow is not white' is exactly about whatever 'Snow is white' is about, say the colour of snow). $t(A \wedge B) = t(A \vee B) = t(A) \oplus t(B)$ (the topic of a junction is the topic of its juncts taken together: 'Arif is tall and handsome', 'Arif is tall or handsome' are both about, say, Arif's height and looks). This makes the extensional connectives topic-transparent (as recommended in Fine 2020, section 2): they add no subject matter of their own to the sentences where they show up. As per the 'fundamental thought' of Wittgenstein 1921 and 1922's *Tractatus*: 'the "logical constants" do not represent' (4.0312). They are about nothing.

Next, take the propositional content of A, what A says, as $|A| = \langle W_A, T_A \rangle$, where W_A is as per the original FDE, i.e., $W_A = \langle |A|^+, |A|^- \rangle$, the pair of truth and falsity sets for A, and $T_A = t(A)$. So $|A| = |B|$, A and B say the same, when they are FDE-equivalent and about the same things. This kicks out precisely the unwelcome Absorption, because what A is about, $t(A)$, in general is not the same as what 'A and/or (A or/and B)' is about, which is $t(A) \oplus t(B)$.

The other FDE equivalences can stay, insofar as their two sides coincide in subject matter. So there's still a difference between AC and topic-sensitive FDE: AC can be set up so that $|A \vee (A \wedge B)| = |A \vee (A \wedge \neg B)|$ fails (again, see Hornischer 2020; Hawke et al. 2024 for details). But it holds in topic-sensitive FDE, because it held in plain old FDE given its Absorption principles: $|A \vee (A \wedge B)| = |A| =$

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$|A \vee (A \wedge \neg B)|$. And the topic constraint does not invalidate it, since the subject matter of the left- and right-side is the same. Who is right, AC or topic-sensitive FDE?

Intuitions may or may not be moot here (do 'Either Arif is tall or he is tall and happy', 'Either Arif is tall or he is tall and unhappy' say the same?). Topic-sensitive FDE is good as it is, if intuitions are not moot, and it seems that that equation should hold. If they are not moot, and it seems that it should fail, what this actually speaks against is the topic-transparency of negation: we need to forfeit $t(\neg A) = t(A)$ and start to assign positive and negative valences to topics. Roughly: a sentence A gets a pair of a positive $t^{+}(A)$ and a negative $t^{-}(A)$ topic, mapped to A occurring in contexts with, respectively, an even or an odd number of negations.

Topic-sensitive FDE is good as it is, if intuitions are moot: simplicity then demands that we stick with plain topics without valences. Uniformity demands transparency for all truth-functional connectives.[12] Besides, one may think that the transparency of negation is a sensible principle anyway. For, as Matteo Plebani reminds me, to disagree is to say opposite things on the same topic. But if transparency fails, when I say '$\neg p$' while you say 'p', we're not quite talking about the same. It seems strange that, whenever we disagree, we are always talking past each other a bit.

On simplicity: one may complain that topic-sensitive FDE is more complicated than AC. Putting all bits together, a topic-sensitive FDE model is $\langle W, T, \oplus, v, t\rangle$: besides the points of evaluation, it features the topic semilattice and two interpretation functions.

But, first, there's a trade-off in complexity here. The FDE truth conditions for $\wedge$ and falsity conditions for $\vee$ are simpler than those of AC. FDE says: 'Junction $A \bowtie B$ is true (false) when both A and B are true (false)'. AC says: 'Junction $A \bowtie B$ is truth(false)made when there's some state truth(false)making A, and some state truth(false)making B, and the current state is a fusion of those'. That's a mouthful.

Second, if one buys certain arguments proposed in Berto 2022, chapter 2, as well as in Hawke et al. 2024, to the effect that truth

[12] Take the Sheffer stroke $A|B$, which, a bit like Sauron's ring ('One connective to rule them all...' — I owe this one to Matteo Plebani, too), can recapture all truth-functions. If you claim all truth-functional connectives are transparent, you'll say that of Sheffer. But if you claim junctions are transparent but negation isn't, what of Sheffer? One could reply that our intuitions of topicality for Sheffer are parasitic on those for negation, since the closest to Sheffer in natural language is probably 'not both'. So Sheffer is not transparent if 'not' isn't.

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conditions and topics are irreducible to each other, and so in particular 'subject matter [is] an independent factor in meaning, constrained but not determined by truth conditions' (Yablo 2014, p. 2), then the 2C semantic setting cuts at the natural joints of meaning: the model has worlds and topics represented separately, and we assign truth conditions and topics via two interpretation functions, v and t, because they *are* distinct and irreducible.

Third, as Levin Hornischer has shown, one can even make FDE better capture subject matters, without directly adding a structure of topics, as in topic-sensitive FDE as described above. One sticks just with the original set W of FDE worlds, but moves up one notch: instead of FDE worlds, one takes sets of FDE worlds as the points at which sentences are evaluated, and so one takes contents as sets of sets of FDE worlds. One can often fine-grain a semantics this way, by moving on up. (E.g., I've done the same in Berto 2010, representing hyperintensional distinctions starting with possible worlds, and looking at sets of sets thereof.) The move refines FDE semantics to the level of AC semantics; in particular, Hornischer 2020, pp. 777–778, proves that Absorption fails again in this setting.

8. *Compatibility*

I left Williamson in the background while discussing truthmaker semantics, but we can now get back to an objection of his, which applies to IWs semantics both in its AC-truthmaking clothing and in its (topic-sensitive) FDE clothing. For it targets bilateralism — here, the idea that we specify propositional contents by giving truth(making) and falsity(making) conditions separately and symmetrically. That's not how we learn language, Williamson objects. It's enough to be given *truth*(making) conditions:

> Imagine that you are being taught a foreign language. Your teacher explains to you exactly what would make a given atomic sentence true. Could you then complain to her: 'You've only done half your job! You've told me what this sentence's *verifiers* are, but you haven't told me anything about its *falsifiers*'? That sounds quite unreasonable. Your teacher has already done enough to enable you to understand the sentence, by normal linguistic standards. You are not missing half its meaning. (2024, p. 132)

Even if it's right that we learn the meanings of atomic sentences by being explained the conditions under which they will have the truth values they can have, I don't think it could work as Williamson has it

in general, for there'll be many cases where giving negative conditions will matter as much as giving positive ones. But I won't quibble over this, for I myself am sympathetic to the idea of giving truth conditions only, rather than truth and falsity conditions separately. To name one thing that has bothered some: in a very symmetric setting like that of FDE, one loses a bit track of what makes of truth, *truth* (besides being so-labeled), rather than falsity (Priest 2008b, p. 54, discusses this issue).

It is well-known that FDE can be equivalently formulated, not as a four-valued semantics but as a two-valued (functional) one, using the so-called Routley Star account of negation. Still same language $\mathcal{L}$; a model is a triple $\langle \mathcal{W}, {}^*, v \rangle$ where the Star * is a period-two operation on worlds ($w^{**} = w$). v at each world w assigns to each atom p in $\mathcal{L}_{AT}$ either the value **T**rue or the value **F**alse. A world w makes true an atom p ($w \Vdash p$) when $v_w(p) = \mathbf{T}$, doesn't ($w \nVdash p$) when $v_w(p) = \mathbf{F}$. This is extended to the whole language via the recursive clauses:

- (S$\neg$) $w \Vdash \neg A$ iff $w^* \nVdash A$
- (S$\wedge$) $w \Vdash A \wedge B$ iff $w \Vdash A$ and $w \Vdash B$
- (S$\vee$) $w \Vdash A \vee B$ iff $w \Vdash A$ or $w \Vdash B$

Negation is a modal: w makes true $\neg A$ when its twin mate w^* doesn't make true A. As Dunn and Restall (2002) have it, the twins are mirror images of each other: where either makes true both A and $\neg A$, the other makes true neither and vice versa. So it makes sense that the star operation be period two: mirroring the mirror takes you back to the original. A world may be its own twin, $w^* = w$. It is then maximally consistent, for $\neg A$ is true at it iff A is not true at it, as per classical negation.

This setting is provably equivalent to the four-valued formulation of FDE (Priest 2008a, ch. 8) and so it gives the same content identities (in particular, that * is period two secures the De Morgan equivalences). The construction adding topics to invalidate Absorption then goes on exactly as before.

One may complain that the symmetry of four-valued FDE has been broken only to restore it just below the surface. The twins w and w^* are themselves symmetric as mirror images: that's what allows the translation with the four-valued setting. One may also complain that the Routley Star clause for negation is not very intuitive.

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But the foundations of the Star semantics, besides being asymmetric, are pretty intuitive. A long time ago Greg Restall (1999) proved that the Star semantics is but a special case of the compatibility semantics for negation (see also Dunn 1993), where one has a primitive compatibility relation C between points and truth conditions are given thus:

- $w \Vdash \neg A$ iff for all w_1 such that wCw_1, $w_1 \nVdash A$

$\neg A$ is true at a world iff A isn't at all worlds it's compatible with. Or equivalently, in terms of primitive exclusion or incompatibility I:

- $w \Vdash \neg A$ iff for all w_1, if $w_1 \Vdash A$ then wIw_1

$\neg A$ is true at a world iff any world where A is true is incompatible with it. The Star semantics is the special case one gets by adding conditions to the basic compatibility semantics, the key one being that each world w has a maximal compatible mate. The Star just maps to it: w^* is a world that's compatible with w and makes true whatever was made true by any compatible w_1.

Compatibility semantics has a venerable history, going back (if not to Plato: see *Sophist*, 257b–c) at least to the Birkhoff-Von Neumann-Goldblatt account of ortho-negation (Birkhoff and Von Neumann 1936; Goldblatt 1974) and being developed by many authors nowadays (see section 2.2. of Horn and Wansing 2025 for a short intro and some references). It is also directly implemented in the frames of relevant logics (e.g., Hornischer and Berto 2025) as well as of substructural logics (e.g., part III of Restall 2000).

More importantly: it's intuitive, because (to recap a story told in Berto and Restall 2018): (a) (in)compatibility is so basic to (our experience of) the world (Kinkaid 2020) that it's a candidate for a primitive notion if anything is. One may root it in our capacity to locate objects in space and time (*this* here, with such and such boundaries, cannot be *that*, down there); or to tell incompatible colours, sizes, shapes — something lots of animals can do, too.

And so (b) we need an exclusion-expressing device to record and communicate incompatibilities, and indeed we have one in all known human languages: negation. For here's how a hypothetical conversation between you and me would go, if we lacked an exclusion-expressing device:

> Me: 'Fred is in the kitchen.' (Sets off for kitchen.)
>
> You: 'Wait! Fred is in the garden.'
>
> Me: 'I see. But he is in the kitchen, so I'll go there.' (Sets off.)
>
> You: 'You lack understanding. The kitchen is Fred-free.'
>
> Me: 'Is it really? But Fred's in it, and that's the important thing. (Leaves for kitchen.).
>
> Your problem is to get me to appreciate that your claims are incompatible with mine. (Price 1990, p. 224)

What we'd need is a: 'Lo, Fred is not in the kitchen'. That is: Fred is somewhere else — in the garden — and his being there rules out his being in the kitchen. The (in)compatibility insight preserves the intuition of asymmetry between positive and negative information, and so between truth and falsity as truth of negation.[13]

Negation picks the weakest incompatible, i.e., what follows from any incompatible: 'The table is not blue' only tells you that the table has some feature or other that rules out being blue (it may be red, yellow, green, ...). In Berto and Restall 2018, we argued for the overall superiority of compatibility semantics with truth conditions only, over one phrased in terms of symmetrical truth and falsity conditions.

And, of course, one can add (in)compatibility as a primitive to truthmaker semantics as well. It has already been done, see, e.g., Plebani et al. 2022. One can come up, in particular, with a unilateralist truthmaker semantics in this setting: state s truthmakes $\neg A$ when it's incompatible with all truthmakers of A and each of its (non-null) parts is incompatible with some. Thomas Randriamahazaka 2025 (p. 237) does precisely that.

9. *Conclusion*

There are many more objections to IWs than those I've had room to discuss here (including further ones by Williamson and Fine). And my replies will be objectionable, too, of course. Nobel Prize Daniel Kahneman once wrote:

[13] A helpful referee of *Crítica* thinks symmetry would be a problem only if it was found at the metaphysical level, for then one would have to postulate (something like) negative facts on a par with positive ones, and negative facts have had a bad press. But I think the idea of asymmetry and information loss in negation is best understood as semantic to begin with.

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> Professional controversies bring out the worst in academics. Scientific journals occasionally publish exchanges, often beginning with someone's critique of another's research, followed by a reply and a rejoinder. I have always thought that these exchanges are a waste of time. Especially when the original critique is sharply worded, the reply and the rejoinder are often exercises in what I have called sarcasm for beginners and advanced sarcasm. The replies rarely concede anything to a biting critique, and it is almost unheard of for a rejoinder to admit that the original critique was misguided or erroneous in any way. On a few occasions I have responded to criticisms that I thought were grossly misleading, because a failure to respond can be interpreted as conceding error, but I have never found the hostile exchanges instructive. (Kahneman 2011, p. 234)

I have a slightly more positive view of critical engagement, at least in philosophy — and what better journal for this than one called *Crítica*? But I also think its usefulness fades away after a few cycles of objections and replies. Some back-and-forth can clarify one's own views, occasionally force one to change and sharpen it. After that, we are often back to the Stalnakerian metric: the success of lasting approaches often depends, rather than on their surviving supposedly deadly objections, on their becoming widespread, entrenched, broadly used and applied in substantive theorizing, perhaps with help from socio-academic luck. By such a metric, IWs are here to stay.[14]

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