Potential versus actual infinity: insights from reverse mathematics

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During April 1–4, 2015 I will serve as an invited Scholar of Consequence with the University of Connecticut Group in Philosophical and Mathematical Logic. As part of that visit I will deliver their Annual Logic Lecture. This document consists of an abstract and references for that lecture.

Abstract

In the philosophy of mathematics, there is a crucial distinction between potential infinity and actual infinity. This distinction gives rise to four contrasting viewpoints: ultrafinitism, finitism, predicativism, and infinitism. I am convinced that of these four, finitism is the most objective. This conviction heightens the importance of Hilbert's program of finitistic reductionism. Some relevant formal systems are PRA, WKL_0, IR, ATR_0, and ZFC. Foundational research over several decades has revealed that large parts of contemporary mathematics, including the applicable parts, can be formalized in systems such as WKL_0 which are finitistically reducible. This seems to provide a possible outline for an objective justification of much of contemporary mathematics.

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