

# THE MODAL LOGIC OF CONFIDENCE

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**ABSTRACT.** In this paper, we construct a modal logic aimed at capturing the degree of doxastic justification offered by a piece of evidence to a proposition. Our *modal logic of confidence*,  $JC$ , is a novel contribution to the family of modal logics known as ‘justification logics’, which originate in Artemov’s [1, 2] logic of proofs,  $LP$ . In  $LP$ , the modal formulas are offered an arithmetical reading as providing witnesses for the implicit existentials in provability logics; so, the modal formula  $[\tau]\varphi$  is intended to be read “ $\tau$  is a proof that  $\varphi$ ”. But, justification logics have also been provided with an epistemic reading to model knowledge with explicit justification under which the modals are read “the agent knows that  $\varphi$  on the basis of the justification  $\tau$ ”. This can be regarded as providing a formal model of belief with doxastic (*cf.* propositional) justification. But, justificatory support comes in degrees and not every proposition that has *some* degree of justificatory support should be believed. Thus,  $JC$  aims to fill this void by capturing the type of justificatory support relevant for the determination of what to believe. So, the modals in  $JC$  take the form  $[\tau]_{\sigma}\varphi$  with the intended reading “ $\tau$  contributes  $\sigma$  justificatory support for  $\varphi$ ”. The graded notion of doxastic support provided by the newly introduced parameter is general in that it only minimally depends on the existence of a partial ordering. At the semantic level, this is interpreted by introducing a bounded lattice into the standard Kripke-Fitting models for justification logic. A second novel aspect of  $JC$  comes with the interpretation of the justification term  $\tau$  found in the modal formulas. Previous work in justification logic took these special terms to be interpreted on the structure provided by the object language. This imposes the requirement that evidence comes in the same form as the propositions that it justifies. However, one might wish to permit non-propositional evidence (such as phenomenal states like feelings, experiences, *etc.*) to contribute to agents’ confidence in propositions. To accommodate this level of generality, the models of  $JC$  include an additional bounded lattice for the interpretation of these justification terms. Thus, our logic  $JC$  exhibits a level of generality suffice to capture an extremely wide range of justificatory relationships that may be relevant to the evaluation of an agent’s beliefs. In addition to the construction of this new logic, we provide a number of preliminary formal results including soundness and completeness. We conclude with a brief discussion of its various applications.

## REFERENCES

- [1] S. N. Artemov. Logic of proofs. *Annals of Pure and Applied Logic*, 67:29–59, 1994.
- [2] S. N. Artemov. Operational modal logic. Technical Report MSI 95–29, Cornell University, 1995.

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