Explanatory and Heuristic Power of Mathematics

13-14 June 2019
Room XI, Villa Mirafiori - Dept. Philosophy - Sapienza University of Rome
Via Carlo Fea 2

Open to the public

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Avigad, Jeremy (*Carnegie Mellon*)
Bangu, Sorin (*Bergen*)
Carter, Jessica (*Southern Denmark*)
Cellucci, Carlo (*Sapienza*)
Colyvan, Mark (*Sydney*)
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UNIVERSITÀ DI ROMA

Sapienza Department of Philosophy
Sapienza Ph.D. Programme in Philosophy

SILFS
società italiana di logica e filosofia delle scienze

Under the auspices of the Italian Society for Logic and Philosophy of Sciences (SILFS)

Program

13 June 2019, room XI

10:30 ***opening***
10:45-11:30 **Mark Colyvan** (Sydney), ***If 13 were not prime: mathematics and counterpossibles***
11:30-11:50 ***discussion***
11:50-12:35 **Emiliano Ippoliti** (Sapienza)**,** ***Mathematical Representations: explanatory and heuristic power***
12:35-12:55 ***discussion***
Chair: Carlo Cellucci

lunch

15:00-15:45 **Jessica Carter** (Southern Denmark), ***Representations and growth of mathematical knowledge***
15:45-16:05 ***discussion***
Coffee
16:20-17:05 **David Danks** (Carnegie Mellon)**,** ***Why are mathematical models useful in cognitive science***
17:05-17:25 ***discussion***
17:25-18:10 **Emily Grosholz** (Penn State), ***Big Data, Mathematical Models, Biological Research, and the Politics of Climate Change***
18:10-18:30 ***discussion***
Chair: Silvia De Toffoli

14 June 2019, room XI

10:30-11:15 **Silvia De Toffoli** (Stanford)**,** ***Grasping How a Proof Supports its Conclusion***
11:15-11:35 ***discussion***
coffee
11:50-12:35 **Jeremy Avigad** (Carnegie Mellon), ***Mathematical rigor and robustness***
12:35-12:55 ***discussion***
Chair: Emily Grosholz

lunch

15:00-15:45 **Marianna Antonutti** (MCMP Munich), ***De Re and De Dicto Knowledge of Mathematical Statements***
15:45-16:05 ***discussion***
Coffee
16:20-17:05 **Carlo Cellucci** (Sapienza), ***Prolegomena to a Heuristic Philosophy of Mathematics***
17:05-17:25 ***discussion***
17:25-18:10 **Sorin Bangu** (Bergen), ***Mathematical explanations of physical phenomena: a minimal characterization (and its problems)***
18:10-18:30 ***discussion***
Chair: Emiliano Ippoliti

20:30 Social dinner

Description

The workshop will investigate the explanatory and heuristic power of mathematics from a conceptual or historical point of view, and their interconnection where possible.

As concerns explanation, we will cover themes like:

- how mathematics provides explanations in other fields—like physics, life science, or the social sciences;
- how mathematics provides intra-field explanations, that is, how pieces of mathematics can be used to provide explanation in other part of mathematics;
- how a philosophical commitment, for instance the nominalism of a theory, affects the explanatory power of that theory.

As concerns heuristics, we will to cover themes like:

- how mathematics is a heuristic engine in other fields—like physics, life science, or the social sciences;
- how mathematics is a heuristic intra-field engine, that is, how certain pieces of mathematics are employed to pose and solve problems in other parts of mathematics (e.g. topology and algebra);
- how a heuristic view of mathematics affects several issues in philosophy of mathematics, e.g. the nature of mathematical objects, or the method of mathematical research.

Abstracts

Marianna **Antonutti** (MCMP Munich), ***De Re and De Dicto Knowledge of Mathematical Statements***

In this talk, I will outline a new approach to the analysis of de re propositional attitudes in mathematics, and more specifically knowledge claims in mathematics, that takes mathematical discourse at face value. A proof will be said to provide *de dicto* knowledge of a mathematical statement if it provides knowledge of a purely existential statement, and to provide *de re* knowledge when it carries additional information concerning the identity criteria for the objects that are proven to exist. I will examine two case studies, one from abstract algebra and one from discrete mathematics, and I will suggest that reverse mathematics can help measuring the 'de re content' of two different proofs of the same theorem, and that the de re/de dicto distinction introduced here lines up with certain model-theoretic properties of subsystems of second order arithmetic. I will argue that the notion of 'de re' content is more general than the notion of constructive content because it applies also to classical contexts, and thus to all informal mathematical reasoning. Finally, I will explore the idea that proofs which provide de re knowledge are *more explanatory* than those that provide merely de dicto knowledge.

Jeremy **Avigad** (Carnegie Mellon), ***Mathematical rigor and robustness***

Of all the demands that mathematics imposes on its practitioners, the requirement that proofs be correct may be the most fundamental. It is also a demand that is hard to fulfill, given the inherent fragility of mathematical proof. This essay some of ways that mathematics supports robust assessment, thereby maintaining coherence and stability.

Sorin **Bangu** (Bergen University), ***Mathematical explanations of physical phenomena: a minimal characterization (and its problems)***

The paper aims to contribute to the clarification of the currently debated notion of a mathematical explanation of a physical phenomenon. I propose what seems like three minimal criteria to be satisfied in order for an explanation to qualify. I also discuss the difficulties encountered when we are to decide whether these criteria are satisfied by concrete examples of this kind of explanations. Upon investigating several such well-known examples, I argue that it is far from clear whether they (fully) meet these desiderata. Time permitting, I'll sketch a new family of examples, and argue that it better satisfies these criteria.

Jessica **Carter** (University of Southern Denmark), ***Representations and growth of mathematical knowledge***

The talk will consider how representations and notations contribute to the development of mathematics. Scholars have come up with various notions in order to explain why certain visual representations are fruitful. Shimojima 1996 shows how graphic representations offer 'free rides'. Manders 2008 notices that new figures 'pop up' as a result of the constructions made in Euclidean diagrams. Macbeth 2014 argues that the possibility of multiple readings of diagrams is partially responsible for the fact that new figures pop up. In the talk I will give examples of fruitful representations used in mathematics, focussing in particular on diagrammatic representations. The intention is to demonstrate that even though information may sometimes be obtained more easily from diagrammatic representations, it does not come for 'free'. In addition, although multiple readings are fruitfully exploited, they do not always lead to new objects popping up.

Carlo **Cellucci** (Sapienza University of Rome), ***Prolegomena to a Heuristic Philosophy of Mathematics***

The aim of this talk is to outline the main features of a Heuristic Philosophy of Mathematics. The latter is not to be confused with the Philosophy of Mathematical Practice, which does not deal with the making of mathematics, because the "practice" to which it refers primarily consists of the propositional products of the activities of mathematicians, as expressed in published work. However, as major mathematicians, from Descartes and Newton to Grothendieck, have forcefully stressed, the creative work of the mathematician is not reflected virtually to any extent in published work. On the other hand, it can be argued that this work can be rationally accounted for. The Heuristic Philosophy of Mathematics aims to give such an account, dealing with the making of mathematics, starting from mathematical discovery. This has immediate implications for issues such as the method of mathematical research or the nature of mathematical objects.

Mark **Colyvan** (University of Sydney, Humboldt Fellow, Ludwig-Maximilians University), ***If 13 were not prime: mathematics and counterpossibles***

Standard approaches to counterfactuals in the philosophy of explanation are geared toward causal explanation. I suggest how to extend the counterfactual theory of explanation to non-causal, mathematical explanation. The core idea here is to model impossible perturbations to the relevant mathematics while tracking the resulting differences to the explanandum (either physical or mathematical, depending on whether we are dealing with extra-mathematical or intra-mathematical explanation). This approach has the potential to provide a unified account of explanation across science, mathematics, and logic.

David **Danks** (Carnegie Mellon), ***Why are mathematical models useful in cognitive science?***

On the one hand, human cognition seems unlikely to be usefully mathematized: it is highly dynamic, context-sensitive, and shaped by vague and imprecise factors. On the other hand, there is a long history in cognitive science of successful (by various measures) mathematical models. In this talk, I will consider several ways to resolve this tension. In particular, I will argue that mathematics serves (at least) two principal functions in cognitive science: first, to help reduce or eliminate scientifically problematic vagueness; and second, to provide a bridge for inter-theoretic constraints that are critical for integrating multiple theories. This case study thus provides insights into the ways that mathematics can prove useful in scientific contexts

Silvia **De Toffoli** (Princeton University), ***Grasping How a Proof Supports its Conclusion***

There are ways of being justified in believing a mathematical claim that are not linked to proofs; for example, via testimony or non-deductive arguments. However, proofs play a central role in the epistemology of mathematics. In my talk, I will focus on what it means to be justified via a proof. I will link being justified with the activity of justifying and I will claim that the appropriate basing relation is put in place by a conscious rational activity: grasping how a proof supports a claim. There are different ways of grasping, which can be divided in at least two broad types: 1) a local, step-by-step grasping and 2) a holistic grasping. These are not mutually exclusive, and often they get combined. Presenting different examples, I will explore how different types of proof facilitate such different types of grasping.

Emily **Grosholz** (Penn State), ***Big Data, Mathematical Models, Biological Research, and the Politics of Climate Change***

My current project concerns the philosophy of biology, using as case studies the projects of my brother Ted Grosholz, a marine biologist at the University of California / Davis who studies invasive species, and of my friend Ruth Geyer Shaw, a population geneticist who with her brother Charles Geyer constructed 'Aster Models' for studying populations of plants on the prairie in Minnesota and Iowa. They are now both working on issues raised by climate change. One of Ted's colleagues is Pamela Reynolds, who is Project Coordinator of the Data Science Initiative, and a good source of information about the growing importance of Big Data in such studies, created, for example, by gene sequencing technology. This leads to a rather philosophical problem: how to distinguish information from noise, and present it to the public in convincing manner? It also raises another problem: how to coordinate different data sets across disciplines? Genbank, the NIH Genetic Sequence Data Base, is a case in point. An increasing output of genetic information makes processing algorithms, collation software, natural language processing and machine learning programs indispensable tools for integrating data across fields. But the conceptual assumptions that organize this technology deserve scrutiny: How should we measure biodiversity or stability in a given area: more generally, how do we assess the health of an ecosystem? Similar issues arise in Ruth's current project studying Partridge pea: it is not a simple matter to quantify how well an organism flourishes in an environment. Her group is studying, empirically and quantitatively, how well the potential rate of adaptation predicts realized adaptation in natural populations, and how a population's genetic variability affects the rate of adaptation over several generations. The scientific import of this research clearly has an ethical and political aspect, for it aims to facilitate assessments of whether populations can adapt rapidly enough to keep up with the pace of climate change.

Emiliano **Ippoliti** (Sapienza), ***Mathematical Representations: explanatory and heuristic power***